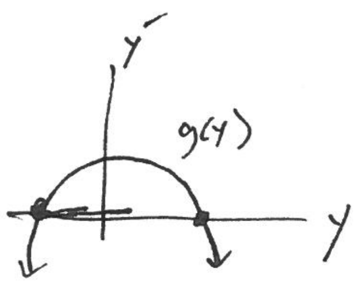
10.5 cont. Qualitative Theory

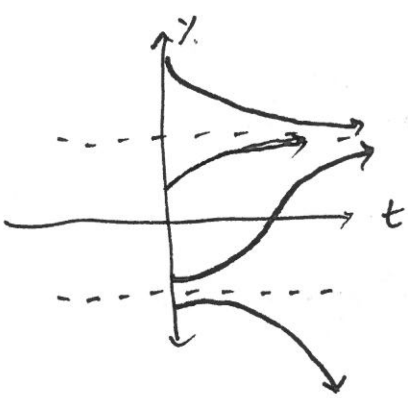
Sketching <u>autonomous</u> D.E.'s

$y' = g(y)$

To sketch solutions to $y' = g(y)$ plot $g(y)$ on yy' plane

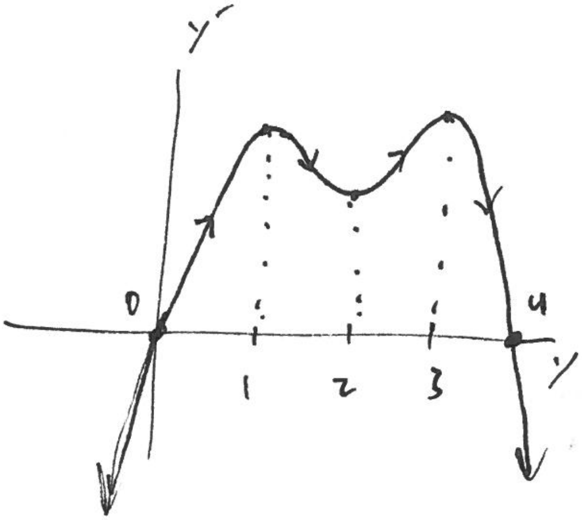


- Find constant solutions : $g(y) = 0$ (where $g(y)$ intersects y-axis) and plot on ty plane
- Find ~~critical~~ max & minimum of $g(y)$ corresponds with inflection changes of solution curves.
- When $y' > 0$ then solution increasing
  when $y' < 0$ solution decreasing



Changes in inflection

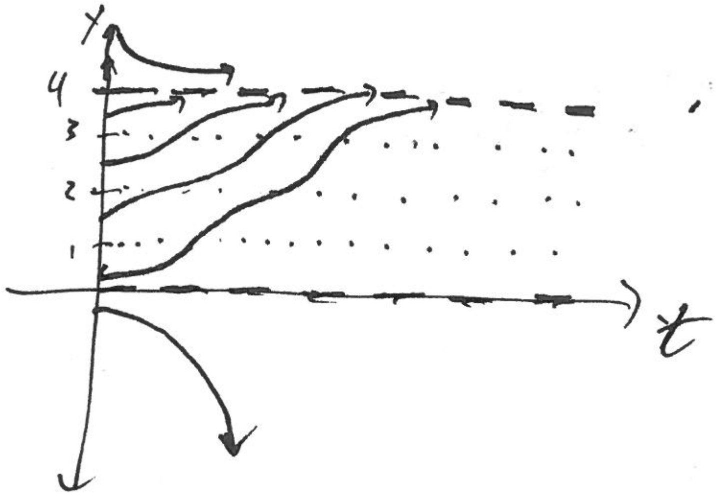
$$y' = (x^2 - 4x)(-x^2 + 4x - 6)$$



constant solutions at $y=0$ and $y=4$

when is it concave up and concave down?

- between $x=0$ & 1 as ~~y increases~~ y increases. y' increases CCU
- between $y=1$ and $y=2$ as $y\uparrow$ $y'\downarrow$ CCD
- between $y=2$ and $y=3$ as $y\uparrow$ $y'\uparrow$ CCU
- between $y=3$ and $y=4$ as $y\uparrow$ $y'\downarrow$ CCD



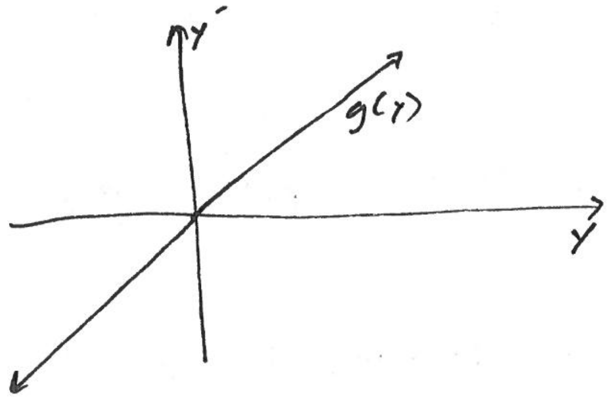
when $y<0$ $y'<0$ so y is decreasing

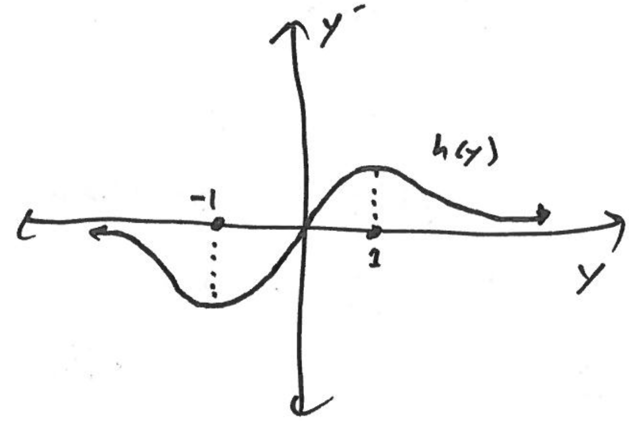
- As ~~[illegible]~~ we follow $y\downarrow$ $y'\downarrow$ so CCD

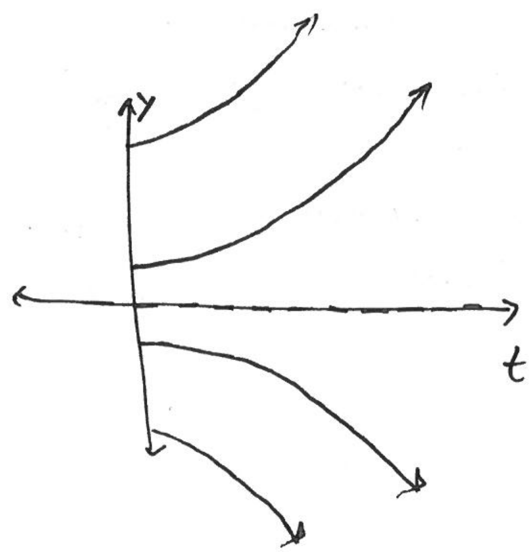
- when $y>4$ $y'<0$ so y is decreasing
- As we follow $y\downarrow$ $y'\uparrow$ so CCU

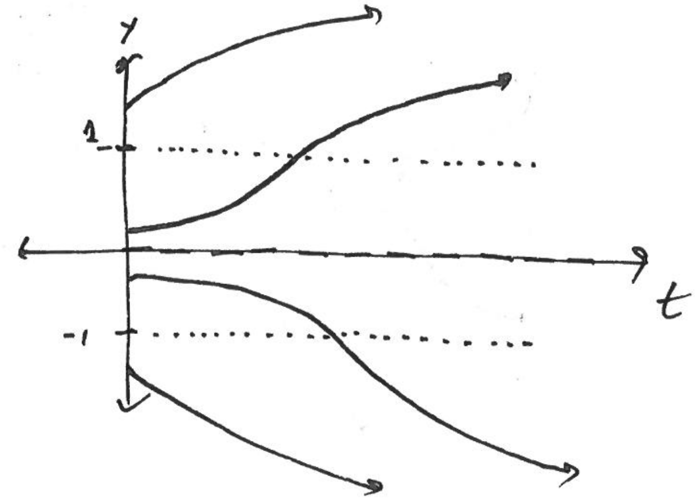
An example

$y' = g(y)$ and $y' = h(y)$









- both have constant solutions

  $y = 0$

- both are increasing when initial value is positive and decreasing when itital value is negative.
- ~~Right~~ Solutions of $y' = h(y)$ change inflection but solutions of $y' = g(y)$ do not
- what initial values give us strictly concave up solutions?

  $y(0) \leq -1$

Plant Growth.

Suppose that a sun flower has a mature height M and its rate of growth is proportional to the product of its height and the difference between its mature height and current height

Recall x & y are proportional if

$$y = kx \quad \text{for some } k.$$

Rate of growth : y'

Height : y

diff between current & mature height : $M - y$ (or $y - M$)

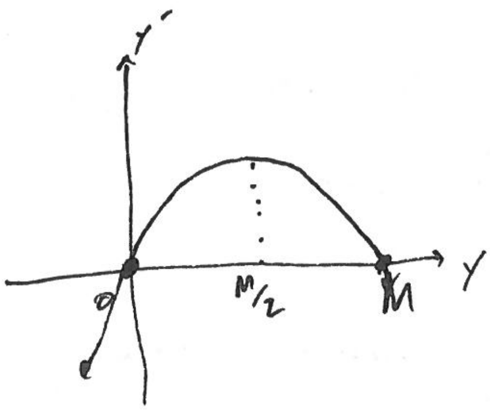
$$y' = ky(M-y) \qquad \text{or} \qquad y' = ky(y-M)$$

k is positive

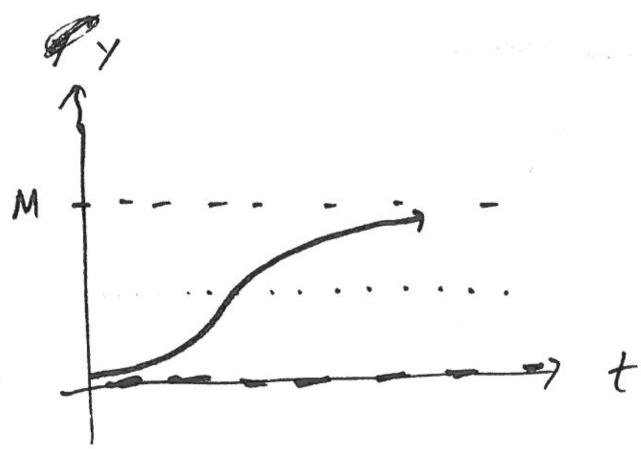
↑ This one is nicer.

k is negative

$$y' = ky(M-y)$$



Constant solutions at $y=0$ and $y=M$



$y' = ky(M-y)$ gives a Logistic growth model

Used to Model

- Population growth
- Disease Spread
- Rumor Spreading

Solution to $y' = ky(M-y)$ is

$$y = \frac{M}{1+Ce^{-Mkt}}$$